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Temperature and frequency effects on fatigue crack growth of uPVC

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Temperature and frequency effects on fatigue crack growth rate have been modelled. The "stress intensity factor"-biased Arrhenius equation and a result from the "two-stage zone" model have been incorporated into the present model. Subsequently, temperature and frequency effects on fatigue crack growth in unplasticized polyvinyl chloride (uPVC) were studied over a temperature range 15–55 °C and a frequency range 0.01–10 Hz. Data for PVC taken from the literature were also included for analysis. It was found that the predicted values from the proposed model are in good agreement with experimental results.

1. Introduction

Polymers are finding increasing use as a material in structural components and a matrix of composite materials. In these applications, structural polymers are often subjected to fatigue loading. Major factors affecting the fatigue crack growth of polymers are known to include temperature and frequency of loading.

The fatigue resistance should decrease as the temperature increases, because chain disentanglement and chain slippage are easier at higher temperatures [1]. Hence, the fatigue crack growth (FCG) rates of polymers, in general, increase with increasing temperature, although some polymers such as polycarbonate (PC) [2] show a somewhat anomalous response in fatigue crack growth rates. For example, fatigue crack growth rates in polystyrene (PS) [3], polyvinyl chloride (PVC) [4], unplasticized polyvinyl chloride (uPVC) [5], acrylonitrile butadiene styrene (ABS) [6], polymethyl methacrylate (PMMA) [1] and polysulphone (PSF) [7], all increase with increasing temperature at a given stress intensity factor range, ΔK.

The fatigue resistance should decrease as the frequency decreases, because the magnitude of chain disentanglement and chain slippage should be larger over a longer period of time (1/frequency). Hence, the FCG rates for polymers such as PMMA [1], PS, PVC and polyphenylene oxide (PPO)/high-impact polystyrene (HIPS) blend, decrease with increasing frequency ([7] p. 84), although exceptions have been reported. One of the reasons for the exceptions is that hysteretic heating at high frequencies softens the material and leads to an higher degree of chain mobility thus increasing the FCG rates [1].

It has been observed in a wide range of polymers that increasing the frequency of loading has the equivalent effect on FCG rate of decreasing the temperature or vice versa, therefore, it is thought that there is a general equivalence between frequency and temperature. In fact, one of the most significant milestones in advancing the knowledge in polymers was the establishment of a relationship between temperature and time effects on the visco-elastic modulus [8]. Similarly, a relationship relating temperature to frequency in the FCG has been required, because such a relationship (a) would enhance the understanding of fatigue mechanisms, influenced by temperature and frequency, and (b) could be used for predicting FCG rates at different temperatures from data of FCG rates at different frequencies or vice versa. A result of an attempt made on this line was the Michel–Manson equation [1, 9, 10] which is a function of four constants and nine parameters including temperature and frequency

$$\frac{da}{dN} = \tau_0^{-1}\frac{c}{f}\exp\left(\frac{-\Delta H_0 + \sigma_{ym}V^{\#}}{RT}\right) \times \frac{(\Delta K - \Delta K_{th})^4}{[\sigma^2(K_c^2 - K_{max}^2)]^p} \tag{1}$$

where τ_0, c, p are constants, f is cyclic frequency, ΔH_0 is the activation energy of the failure process. $V^{\#}$ is the activation volume, $R(=8.31\ \mathrm{J\,mol^{-1}\,K^{-1}})$ is the gas constant, ΔK and ΔK_{th} are the applied stress intensity range and threshold range, respectively, σ is the yield strength, $\sigma_{ym} = \sigma(1 + r)/2$, r is the ratio of minimum to maximum cyclic load, K_c is the fracture toughness and K_{max} is the stress intensity factor at maximum cyclic load. However, this equation, which is basically a combination of the Zhurcov equation [11] and an empirical equation [12], has never been fully shown to agree with experimental results, perhaps because the parameters such as σ, K_c and ΔK_{th} are not independent of temperature and frequency.

This paper presents a new model of FCG rates subjected to variations of temperature and frequency, and demonstrates an applicability to uPVC and PVC [4].

2. Theory

2.1. Frequency effect

The time, t, dependence for polymers may be expressed as [13]

$$E = E_0 t^{-n} \quad (2)$$

where E is the tensile creep modulus, E_0 is the unit time modulus, and $n = \mathrm{d}\ln E/\mathrm{d}\ln t$. Although Marshall *et al.* [13] indicated that n decreases at extremes of rate or temperature, the constant, n, is assumed to be approximately constant in a certain range for any visco-elastic process. Williams [14] related n to frequency, f, based on the line-zone model by the following relationship

$$\frac{\mathrm{d}a}{\mathrm{d}N} \propto f^{-nm} \quad (3)$$

where m is the Paris power-law exponent which is insensitive to temperature and frequency for many polymers (see Equation 8) so that it is approximately constant.

2.2. Temperature effect

Recently, Kim and Mai [5] suggested the following equation to account for the effect of temperature on FCG rates ($\mathrm{d}a/\mathrm{d}N$) in uPVC and other polymers

$$\frac{\mathrm{d}a}{\mathrm{d}N} = B\exp\left[\frac{-(\Delta H_{\mathrm{th}} - \gamma\log\Delta K)}{RT}\right] \quad (4)$$

where B is a constant, ΔH_{th} is the apparent activation energy independent of ΔK, γ is a constant, $\gamma\log\Delta K$ is a "stress intensity factor"-biased term, R is the gas constant and T is the absolute temperature. The ΔH_{th}, expresses the effect of fatigue mechanism under a given condition.

2.3. Synthesis development

In order to accommodate the two test variables, temperature and frequency, in one equation, the following procedure was conducted. Taking logs on Equation 3 gives

$$\log\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right) \propto -nm\log f \quad (5)$$

Accordingly, a family of straight lines with a slope of $-nm$ for a given ΔK, one line for each temperature, can be obtained in a plot of $\log(\mathrm{d}a/\mathrm{d}N)$ against $\log f$ as shown in Fig. 1.

Because FCG rates influenced by temperature at a given frequency can be described by Equation 4, it allows us to relate frequency to temperature by

$$-nm = \frac{\log a_T}{\log a_f} \quad (6)$$

where a_T is

$$a_T = \frac{(\mathrm{d}a/\mathrm{d}N)}{(\mathrm{d}a/\mathrm{d}N)_{\mathrm{r}}} = B\exp\left[\frac{-(\Delta H_{\mathrm{th}} - \gamma\log\Delta K)}{RT}\right] \Bigg/ \left(B\exp\left[\frac{-(\Delta H_{\mathrm{th}} - \gamma\log\Delta K)}{RT}\right]\right)_{\mathrm{r}}$$

and $a_f = f/f_{\mathrm{r}}$. The subscript, r, denotes an arbitrarily chosen reference point in the coordinate system.

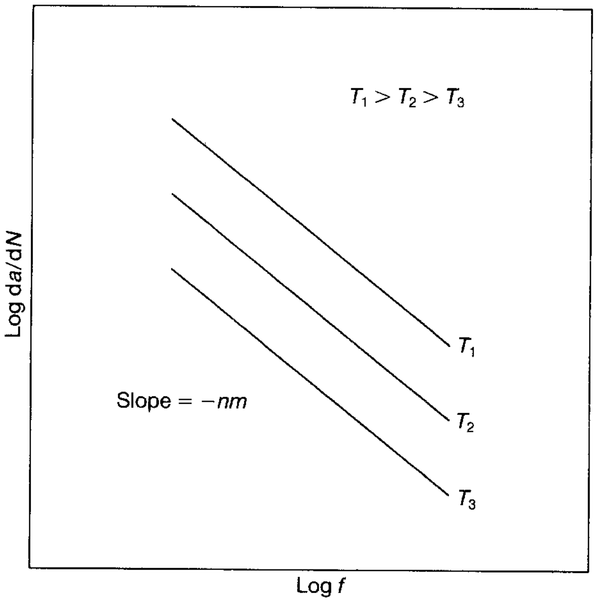

Figure 1 Schematic variation of $\mathrm{d}a/\mathrm{d}N$ with frequency at a given ΔK and given temperatures T_1, T_2 and T_3.

Therefore, we obtain FCG rate, $\mathrm{d}a/\mathrm{d}N$, as

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \left(\frac{f}{f_{\mathrm{r}}}\right)^{-nm}\left(B\exp\left[-\frac{\Delta H_{\mathrm{th}} - \gamma\log\Delta K}{RT}\right]\right)_{\mathrm{r}} \quad (7)$$

This equation expresses the effects of frequency and temperature.

3. Experimental procedure

The uPVC pipes were taken from normal production 150 mm class 12 (wall thickness ~9 mm) and they were made from unmodified, lead-stabilized PVC resin with a K value of 67 corresponding to an approximate molecular weight average of 170 000. The pipes showed no attack on immersion in methylene chloride for 20 min at 20 °C and were thus considered well processed. Methylene chloride temperature (MCT) [15] for 20 min was found to be 24 °C. The pipes were slit and warmed in an oven at 100–105 °C for 20 min before being opened up and pressed between metal plates to produce a flat sheet. Single-edge-notched (SEN) samples 70 mm × 210 mm were cut from the flattened sheets so that the crack would propagate in the pipe extrusion direction.

Crack growth was monitored by a computerized data acquisition system using the principles developed by Mai and Kerr [16]. The system consists of a conductive surface grid printed on a specimen interfaced to a personal computer and an MTS closed-loop servo-hydraulic fatigue testing machine with a load capacity of ±1 tonne. A temperature chamber with an accuracy of ±1 °C was used to control fatigue test temperatures: subambient temperatures were controlled in a flowing nitrogen gas environment. The tests were conducted under constant load range conditions with a sinusoidal load wave form and a stress ratio of 0.2. In order to minimize the hysteretic heating effect, the highest test frequency used was limited to 10 Hz (Cheng *et al.* [1] measured the temperature at the crack tip of a PMMA for a frequency of 10 Hz and found that there was only a negligible change in temperature at the tip).

4. Results

The experimental data of FCG rates are plotted in accordance with the Paris power law

$$\frac{da}{dN} = A\,\Delta K^{m} \qquad (8)$$

where A and m are constants, and ΔK is the stress intensity factor range. FCG rate data obtained at

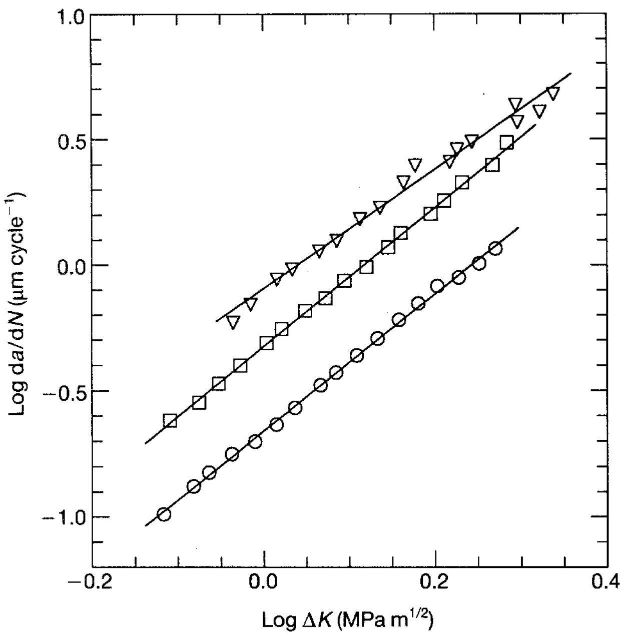

Figure 2 Influence of frequency on fatigue crack growth rate of uPVC at 55 °C. (▽) 0.1 Hz, (□) 1 Hz, (○) 10 Hz.

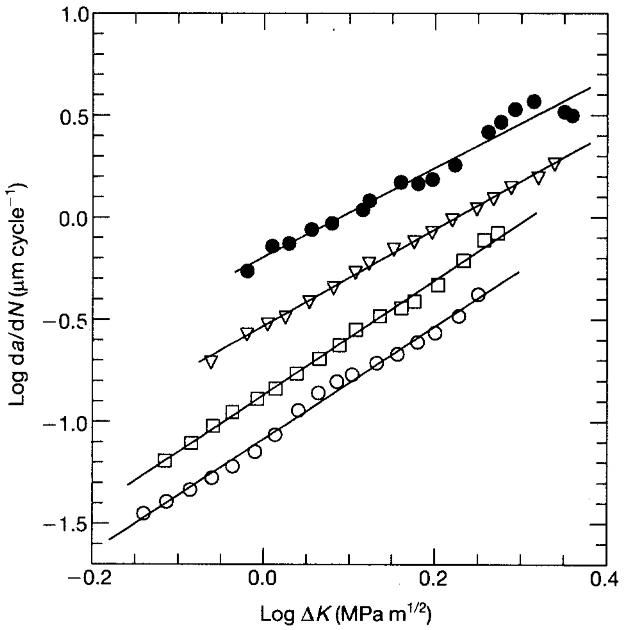

Figure 3 Influence of frequency on fatigue crack growth rate of uPVC at 35 °C. (●) 0.01 Hz, (▽) 0.1 Hz, (□) 1.0 Hz, (○) 10 Hz.

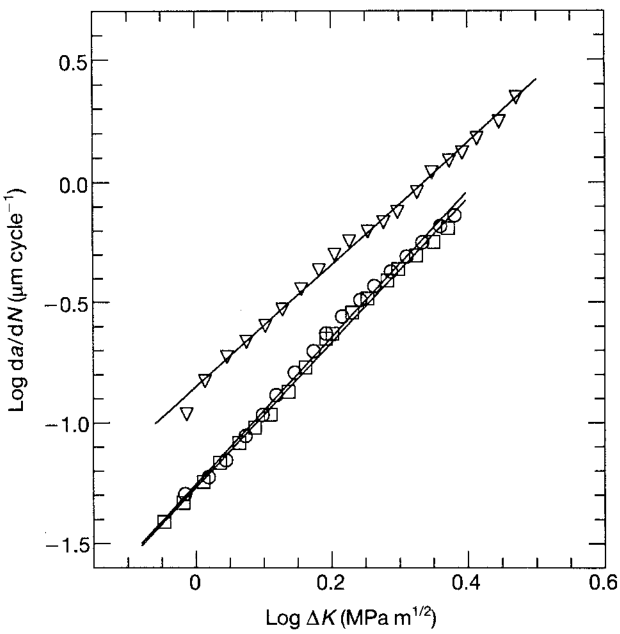

Figure 4 Influence of frequency on fatigue crack growth rate of uPVC at 15 °C. (▽) 0.1 Hz, (□) 1 Hz, (○) 10 Hz.

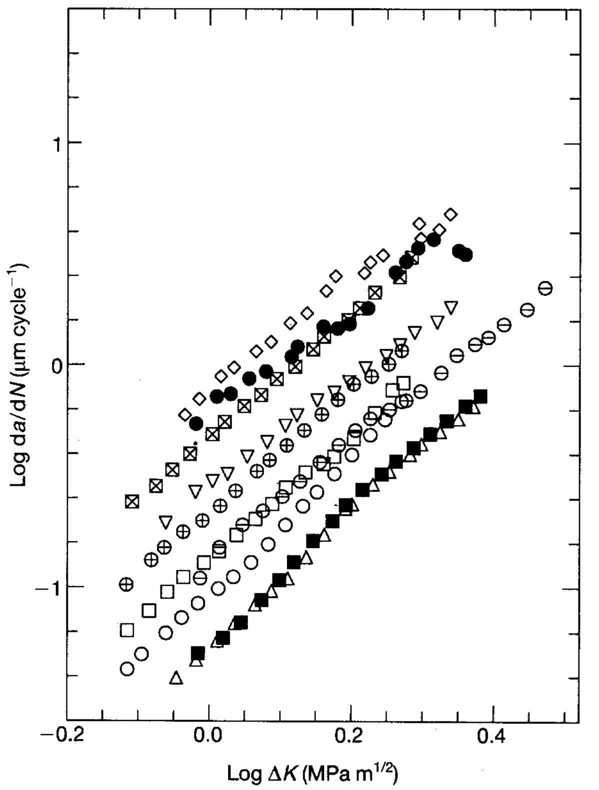

Figure 5 A combined plot of data from Figs 2, 3 and 4. Temperature: (◇, ⊠, ⊕) 55 °C, (●, ▽, □, ○) 35 °C, (⊖, △, ■) 15 °C. Frequency: (●) 0.01 Hz, (◇, ⊖, ▽) 0.1 Hz, (⊠, △, ▽) 1 Hz, (⊕, ■, ○) 10 Hz.

55 °C with a frequency range of 0.1–10 Hz, at 35 °C with a frequency range of 0.01–10 Hz and at 15 °C with a frequency range of 0.1–10 Hz are shown in Figs 2–4, respectively. Generally, the FCG rates are seen to be sensitively influenced by test frequency, except for the data at 15 °C and 10 Hz (Fig. 4) which are the lowest temperature and the highest frequency, respectively. A combined plot of data from Figs 2–4 is

TABLE I Values of Paris power-law constants obtained from experiments, A and m of $da/dN = A\Delta K^m$(da/dN in μm cycle^{-1}, ΔK in MPa m$^{1/2}$)

Temperature (°C)	Frequency (Hz)	A	m
15	0.1	0.141	2.54
	1	0.054	3.01
	10	0.055	3.06
35	0.01	0.638	2.19
	0.1	0.294	2.36
	1	0.136	2.80
	10	0.081	2.76
55	0.1	0.804	2.38
	1	0.472	2.77
	10	0.219	2.72

shown in Fig. 5 to display both temperature and frequency effects. The values of A and m obtained from the experiments are given in Table I.

5. Analysis and discussion

In order to check if there is any fatigue mechanism change in uPVC, we examined the fracture surfaces at $\Delta K = 1$ MPa m$^{1/2}$. The features of the fracture surfaces did not seem to be affected by the variations of temperature and frequency, which indicates that there is no fatigue mechanism change. Fig. 6 shows a typical fracture surface taken at $\Delta K = 1$ MPa m$^{1/2}$ (35 °C, 0.1 Hz). We also examined fracture planes which are perpendicular to fracture surfaces and found that crazing is generally the predominant deformation mechanism for all the temperatures and frequencies, except for 15 °C and 10 Hz. Thus, it is indicated that at the 15 °C and 10 Hz, a transitional behaviour of fatigue mechanism occurs. The contradiction between observations of fracture surfaces and planes (at 15 °C, 10 Hz) is perhaps due to lack of manifestation at the onset of transition. The transitional behaviour of uPVC for the same grade but different batch of pipes is well described elsewhere [5]. The data for 15 °C and 10 Hz were not taken into consideration for the following analysis.

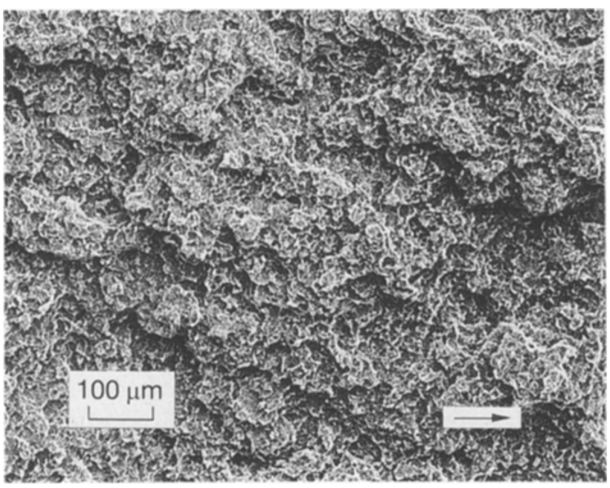

Figure 6 Fracture surfaces of uPVC at $\Delta K = 1$ MPa $m^{1/2}$ for 35 °C, 0.1 Hz. Arrow indicates the crack growth direction.

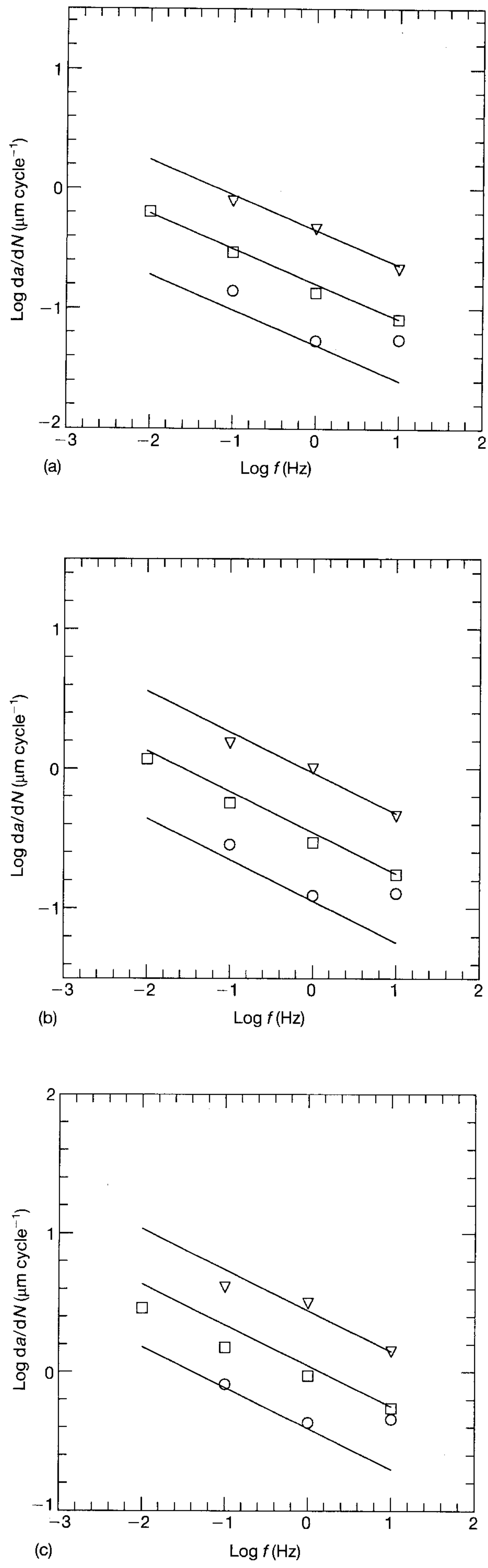

Figure 7 Variation of da/dN of uPVC with frequency at (▽) 55 °C, (□) 35 °C and (○) 15 °C: (a) at $\Delta K = 1.00$ MPa $m^{1/2}$, (b) at $\Delta K = 1.32$ MPa $m^{1/2}$ and (c) at $\Delta K = 2.00$ MPa $m^{1/2}$.

The FCG rates of uPVC at $\Delta K = 1$ ($= \log 0$), 1.32 ($= \log 0.12$) and 2.00 ($= \log 0.3$) MPam$^{1/2}$ are plotted in Fig. 7a–c respectively, to show the frequency effect at different temperatures. An average value of slopes ($-nm$) was found to be -0.295 and is plotted in Fig. 7 using Equations 4 and 5. The value of $-nm$ is well fitted to the data other than the data points at 15 °C and 10 Hz. Equation 7 is now qualified to be used with a constant value of $-n$ — for predicting FCG rates influenced by both temperature and frequency. If we choose $f_r = 1$ Hz for a reference point, Equation 7 becomes

$$\frac{da}{dN} = f^{-nm}\left(B\exp\left[-\frac{\Delta H_{th} - \gamma\log\Delta K}{RT}\right]\right)_r \quad (9)$$

where B, ΔH_{th} and γ for 1 Hz can be determined from Equations 4 and 8, i.e.

$$m = \frac{\gamma}{RT\ln 10} \quad (10)$$

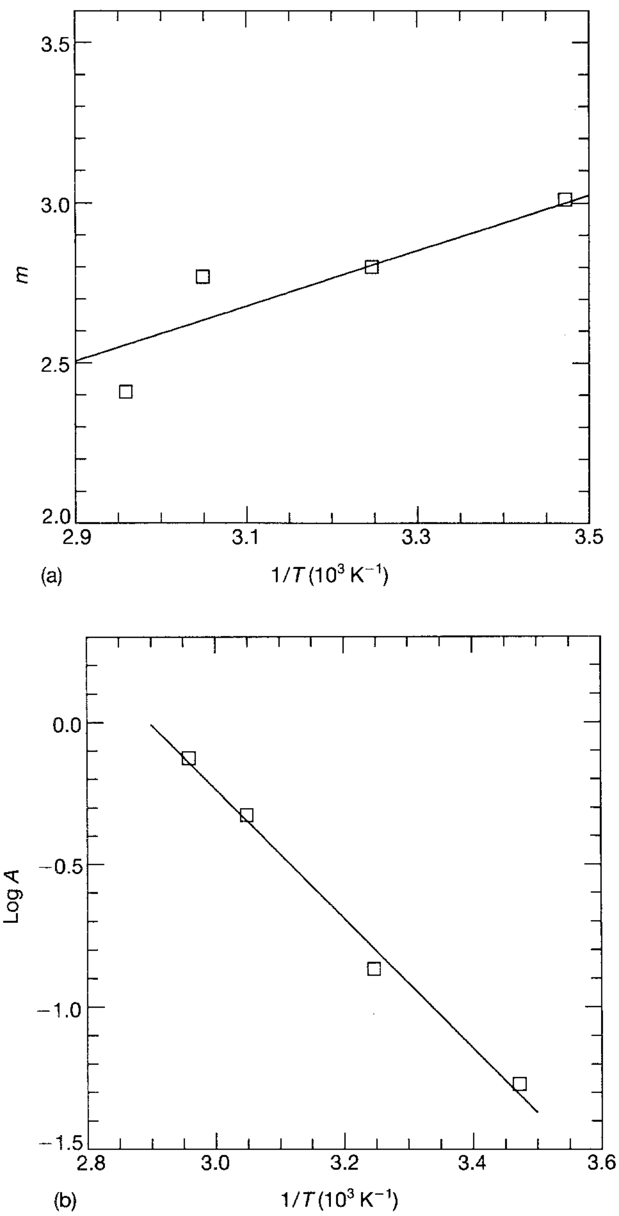

Figure 8 Plots for uPVC at 1 Hz: (a) Paris power-law exponent m versus $1/T$, and (b) Paris power-law constant A versus $1/T$.

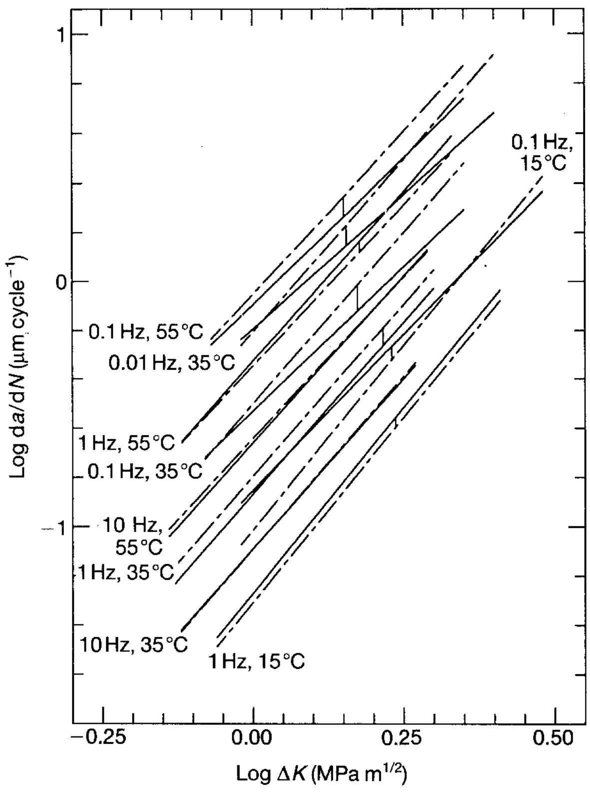

Figure 9 Comparisons of (— - —) predicted values with (——) experimental results for uPVC.

TABLE II Predicted values of Paris power-law constants obtained from Equation 9, A, and m of $da/dN = A\Delta K^m$ (da/dN in μm cycle^{-1}, ΔK in MPa m$^{1/2}$).

Temperature (°C)	Frequency (Hz)	A	m
15	0.1	0.098	3.00
	1	0.049	3.00
	10	0.025	3.00
35	0.01	0.622	2.81
	0.1	0.316	2.81
	1	0.160	2.81
	10	0.081	2.81
55	0.1	0.888	2.63
	1	0.450	2.63
	10	0.228	2.63

and

$$\log A = \log B - \frac{\Delta H_{th}}{RT\ln 10} \quad (11)$$

Fig. 8a and b shows plots in accordance with Equations 10 and 11, respectively, and values of B, ΔH_{th} and γ at 1 Hz were found to be 3.752×10^6, 43.6 kJ mol^{-1} and 16.54, respectively. (Values of constants were determined for da/dN in μm cycle^{-1}, ΔK in MPa m$^{1/2}$, R in kJ mol^{-1} K^{-1} and ΔH_{th} in kJ mol^{-1}). When Equation 9 is used in conjunction with these obtained parameter values, we can predict da/dN versus ΔK. Fig. 9 shows comparisons of predicted da/dN with those of experiments. Also, the predicted values of A and m of Equation 8 are shown in Table II. Comparisons of these values (Table II) with those of experiments given in Table I show good agreement with each other.

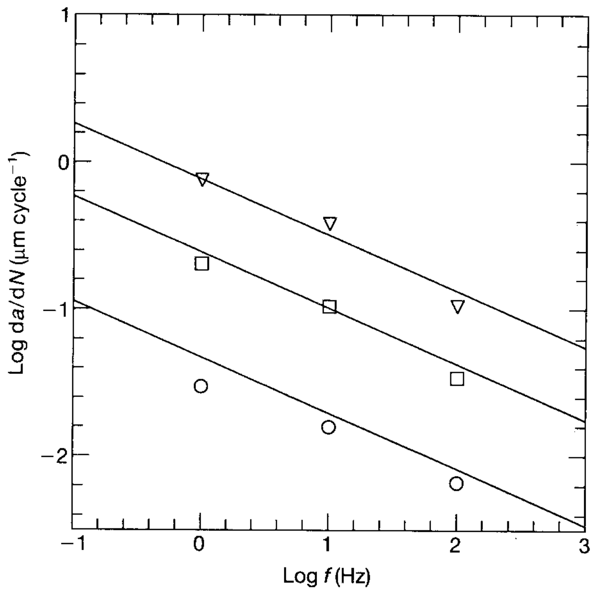

Figure 10 Influence of frequency on fatigue crack growth rate of PVC [4] at 22 °C for $\Delta K = (\triangledown)$ 1.00, (□) 0.65 and (○) 0.35 MPa $m^{1/2}$ $nm = 0.381$.

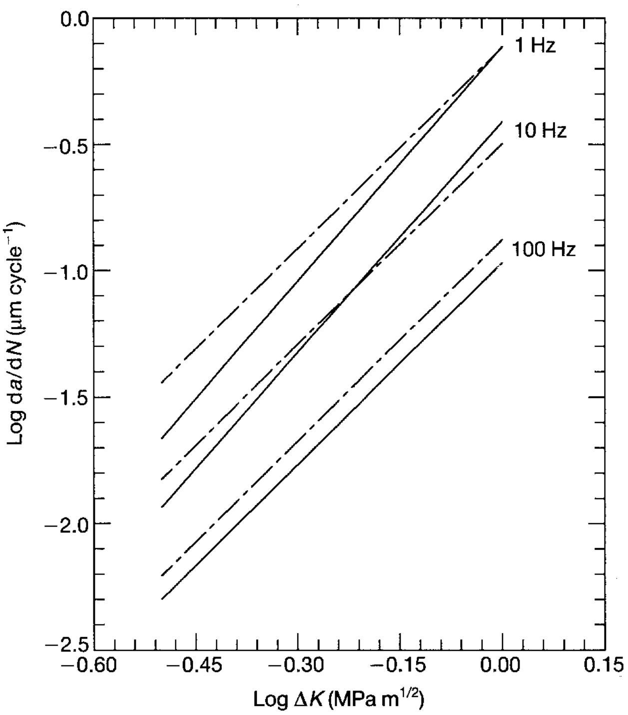

Figure 11 Comparisons of (—·—) predicted values with (——) experimental results for PVC [4].

6. Analysis of fatigue crack growth data of PVC

Because the analysis based on Equation 7 has shown to be useful for predicting the FCG rates of uPVC, the analysis was extended to the data for PVC taken from Phillips *et al.* [4]. The FCG rates of PVC at $\Delta K = 1$, 0.65 and 0.35 MPa m$^{1/2}$ are plotted in Fig. 10 to show frequency effect at different values of ΔK. An average value of slopes ($-nm$) was found to be -0.381 and is plotted in Fig. 10 in accordance with Equations 4 and 5. The values of ΔH_{th}, B and γ in Equation 4 were obtained from earlier work [5] and the same procedure described in the previous section was employed. As a result, comparisons of predicted values with experimental results are plotted in Fig. 11. The predictions are in good agreement with experiments.

7. Conclusion

The model proposed here has been demonstrated to be useful for predicting the effects of temperature and frequency on FCG rates. Predicted values of FCG rates from the model are found to be in good agreement with experimental results of uPVC and PVC [4].

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